

EASSS 2023 Recommender Systems

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Recommender Systems

"**Recommendation:** a suggestion or proposal as to the best course of action, especially one put forward by an authoritative body."

- Oxford Languages

" **System:** 1 . a set of things working together as parts of a mechanism or an interconnecting network; a complex whole. *2. a set of principles or procedures according to which something is done; an organized scheme or method."*

- Oxford Languages

Recommendation

- Recommendations have existed since ancient times.
- Societies have benefited from following the advice of their ancestors.
- For example, priests have traditionally provided guidance to people.
- However, these examples are not based on systematic tools.

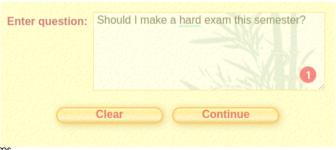
What is considered the first systematic tool created to provide advice to people about their future?

Fu Xi

- Fu Xi is a mythical emperor of China
- He taught his people how to domesticate animals, cook, fish with nets, and hunt with iron weapons over 5000 years ago.
- He is the author o I Ching (*The book of changes*), one of the oldest book in the world.
 - Guide for moral living but also a personalised oracle for one's future!
 - Instead of machine learning, he used patterns in nature and sky.

I Ching

- You can do your I Ching consult online
- I did one, let's see... I was in doubt if the exam for this course should be hard or easy. Then, I asked for these divine recommender what to do:



I Ching - Hexagram



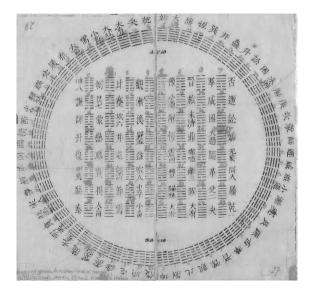
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5

I Ching - Recommendation

	Cast Hexagram:
	55 - Fifty-Five
	Fêng / Abundance
	Thunder and Lightning from the dark heart of the storm: The Superior Person judges fairly, so that consequences are just.
	The leader reaches his peak and doesn't lament the descent before him. Be like the noonday sun at its zenith. This is success.
Cast Hexagram: 55	SITUATION ANALYSIS: You are in a position of authority in this situation. Archetypally, you are the New King, returned from your quest to claim your thron
	However, you are enlightened enough to realize that you are merely a part of a cyu and that you must someday yield your throne to the new kid in town, the younge faster gunslinger, the young turk, the next returning hero, the next New King. Fretting about the inevitable descent is senseless.
	For now you must play your role to the hilt and use this gift of power to govern yo world as best you can. You are the best person for the job. That's why you were chosen. Give it your personal best.

I Ching



Nowadays...



Nowadays...



Why recommender systems?



It is not just about youtube...

- 3,000,000 books per year (Total of 129,864,880 available)
- 137 million new tracks every year
- 500 million tweets are shared every day
- You have 38 million minutes of live
- Conclusion: we need some help

Recommender Systems

- Recommenders recommend:
 - Items to users (most common)
 - Users to Items
 - Items to Items
 - Users to Users
- Items are movies, products, news, music, books, recipes, ...

Working in pairs: try to find one example of each four recommender scenarios above.

Recommender Systems

- WLOG, we will focus on recommend users relevant items
 - Predictive modelling: predict the rate of item m by the user u
 - Retrieve modelling: learning a rank systems
- Typically based on past interactions and (or) attributes (from users and items)
- Interactions: normally modelled as interaction matrix
 - Explicit: a user rate a song with 4 stars in a scale from 0 to 5
 - Implicit: a user watched 80% of a movie
- Attributes: normally modelled as attribute matrices
 - Users: gender, age, location, ···
 - Items: text, video, meta-data, ···
- 13 Recommender Systems EASSS 2023

Modelling Interactions: Explicit feedback

	m 1	m 2	 mn
U1	?	2	 3
U2	5	1	 ?
Из	?	3	 1
Um	4	4	 ?

Modelling Interactions: Implicit feedback

U1	m1	12/11/2021 09:01:21	Watch	25%
U 2	m1	17/03/2021 14:27:09	Clicked	
U 2	m 4	17/03/2021 14:22:09	Clicked	Purchase
Um	mn	14/06/2020 23:14:46	Watch	100%

Modelling Interactions: Implicit feedback

U1	m1	12/11/2021 09:01:21	Watch	25%
U2	m ₁	17/03/2021 14:27:09	Clicked	
U2	m 4	17/03/2021 14:22:09	Clicked	Purchase
Um	mn	14/06/2020 23:14:46	Watch	100%

	m ₁	m 2	 mn
U 1	1	0	 0
U 2	1	0	 0
Uз	0	1	 1
Um	0	1	 1

Modelling Attributes



Modelling Attributes

	Color	Price	Category
mı	Black	24936	Chair
m ₂	Red and White	24944	Chair
mз	Red and White	1299	T-Shirt
m4	Black	1104	Chair



Personalised Machine Learning

- Personalisation is not a simple regression or classification problem
- A personalised model implies that: if the user have different interactions (or attribute) the recommendation should be different
- Suppose the vector a_u (a_m) are attribute vectors of user u (item m)
- We can use linear regression to predict how users u will like item m

$$r_{ui} = \omega^ op imes \begin{bmatrix} a_u \\ a_m \end{bmatrix}$$

- Is linear regression a personalised model for recommenders? No!
- 17 Recommender Systems EASSS 2023

Recommendation Algorithms

Collaborative Filtering



Day One: Joe and Julia independently read an article on police brutality



Day Two: Joe reads an article about deforestation, and then Julia is recommended the deforestation article

Content-Based Filtering



Day One: Julia watches a Drama

 $\mathbf{1}$

DRAMA



Day Two: Dramas are recommended

Recommender as a Matrix

- As we saw, we can model recommenders as matrices.
- The ratings can be stored in a ranking matrix R of dimension m × n with elements from ℝ ∪ {?}.
- An example of a rating matrix for m = 4 users and n = 6 items can read

$$R=egin{pmatrix} 1&?&?&2&?&1\ ?&2&3&?&2&1\ 1&5&5&?&?&5\ ?&?&2&?&3\ \end{pmatrix}$$

Meaning, e.g., that user u_1 ranked items i_1 and i_6 with 1 star, item i_4 with 2 stars and had no interactions with items i_2 , i_3 and i_4 .

- Our goal is to predict the unknown ratings $r_{u,i} = ?$ using the knowledge of the known ratings $r_{u,i} \neq ?$.
- 18 Recommender Systems EASSS 2023

Idea of matrix factorization

• By **matrix factorization** we usually mean expressing a given matrix *R* as a matrix product of two (or more) matrices with some non-trivial properties. For example:

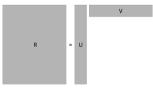
$$R = UV^{ op}$$

• These factorizations are a cornerstone of many algorithms and methods or are used to reach more numerically stable computations.

Do we need to know all the entries of a matrix R to factorize it, for example $R = UV^{\top}$?

Intuition of matrix factorization

- As for the recommendation systems, the inspiration comes mainly from the SVD as it can be used for constructing latent features or, in other words, dimensionality reduction using projections to lower dimensional space.
- The very basic idea of the lower dimensional approximation of an input matrix R of dimension $m \times n$ is based on this first-linear-algebra-lesson fact: Multiplying matrices U of dimension $m \times d$ and V of dimension $d \times n$ we get a matrix of dimension $m \times n$. This is true for any positive integer d.
- And this is the idea: Given a rating matrix *R*, find lower dimensional matrices *U* and *V* so that the known elements of *R* are well approximated by the matrix **UV**[⊤].



Matrix factorization for recommenders

- R ~ U
- Let us denote:
 - The *i*-th row of *U* as u_i ; the number of rows of *U* equals the number of users $|\mathcal{U}|$.
 - The *j*-th column of V as v_j ; the number of columns of V equals the number of items $|\mathcal{I}|$.
 - Ω the subset of $\mathcal{U} \times \mathcal{I}$ of user-item pairs (i, j) such that $r_{i,j}$ is known, i.e., $r_{i,j} \neq ?$.
- The approximation of $r_{i,j}$ is given by the number $u_i^T v_j$, i.e., by the dot product of the two *d*-dimensional vectors.

Optmization Problem

• The **error of approximation** is usually measured by the squared residual:

$$(r_{i,j}-u_i^Tv_j)^2.$$

• Hence, the matrices *U* and *V* are obtained by solving the optimization task:

$$\operatorname{argmin}_{\mathbf{U},\mathbf{V}} \sum_{(i,j)\in\Omega} (r_{i,j} - u_i^T v_j)^2 + \lambda (\sum_x ||u_x||^2 + \sum_y ||v_y||^2).$$

Sparsity and prediction

- The matrices *U* and *V* are optimized only by considering the known entries of *R* that are usually only a minority of entries.
- E.g. in the Netflix prize in 2006 there were n = 17K movies and m = 500K users, meaning that the matrix R had 8500M entries. But only 100M was given by Netflix!
- Still, the result of the matrix multiplication UV^{\top} is a matrix having the same dimensions as R with all entries known!
- The unknown rating $r_{i,j} = ?$ is estimated as $\hat{r}_{i,j} = u_i^T v_j$.

Example

• Consider our toy example matrix from above:

$$R = \begin{pmatrix} 1 & ? & ? & 2 & ? & 1 \\ ? & 2 & 3 & ? & 2 & 1 \\ 1 & 5 & 5 & ? & ? & 5 \\ ? & ? & 2 & ? & ? & 3 \end{pmatrix}.$$

- Assume that we chose the hyperparameter d = 2, i.e., we look for approximation matrices U and V with dimensions 4×2 and 2×6 , respectively.
- Let us pretend that the matrices resulting from the optimization are

$$U = \begin{pmatrix} 0.3 & 0.7 \\ 0.3 & 0.5 \\ 0.2 & 0.4 \\ 0.2 & 0.1 \end{pmatrix} \quad \text{and} \quad V^{\top} = \begin{pmatrix} 1 & 10 & 11 & 10 & 4 & 20 \\ 1 & -1 & -2 & -1 & 1 & -4 \end{pmatrix}.$$

Example

• The resulting approximation is

$$\mathbf{U}\mathbf{V}^{\top} = \begin{pmatrix} 0.3 & 0.7\\ 0.3 & 0.5\\ 0.2 & 0.4\\ 0.2 & 0.1 \end{pmatrix} \begin{pmatrix} 1 & 10 & 11 & 10 & 4 & 20\\ 1 & -1 & -2 & -1 & 1 & -4 \end{pmatrix} = \\ = \begin{pmatrix} 1 & 2.3 & 1.9 & 2.3 & 1.9 & 3.2\\ 0.8 & 2.5 & 2.3 & 2.5 & 1.7 & 4\\ 0.6 & 1.6 & 1.4 & 1.6 & 1.2 & 2.4\\ 0.3 & 1.9 & 2 & 1.9 & 0.9 & 3.6 \end{pmatrix},$$

where the red numbers are the desired predictions!

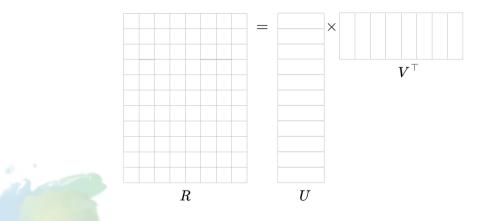
- E.g. the 3rd user predicted rating of the 4th item is $\hat{r}_{3,4} = 1.6$.
- 25 Recommender Systems EASSS 2023

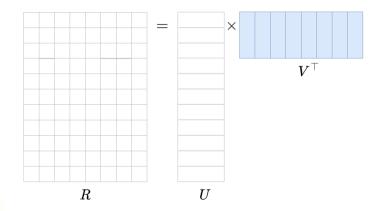
Supervised learning task

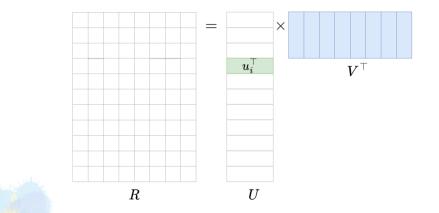
- The learning parameters: $U \in \mathbb{R}^{m imes d}$ and $V \in \mathbb{R}^{n imes d}$
- The hyperparameters:
 - the regularization constant $\lambda > 0$,
 - the matrix dimension d, which is a positive integer (significantly smaller than $\min\{m, n\}$).
- These hyperparameters can be tuned in the usual way via crossvalidation
- Therefore we would like to learn U and V, given d and λ by

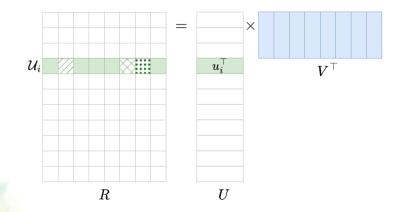
$$\operatorname{argmin}_{\mathbf{U},\mathbf{V}} \sum_{(i,j)\in\Omega} (r_{i,j} - u_i^T v_j)^2 + \lambda (\sum_x ||u_x||^2 + \sum_y ||v_y||^2).$$

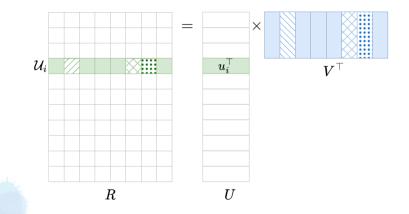
- The idea of ALS is to fix alternately the matrix *U* and *V*. The non-fixed matrix is then considered learning variable and a subject to minimization.
- With one of the matrices fixed, the optimization problem becomes convex and very similar to the linear regression problem.
- Let's try ti understand how the mechanism works

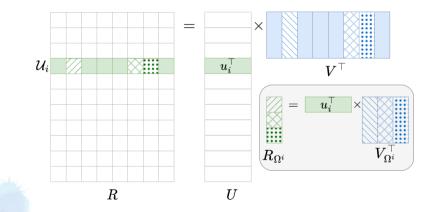


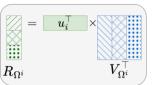












• Then we have the following optimization problem

$$\min_{u_i} ||R_{\Omega^i} - u_i^{\top} V_{\Omega^i} \top ||^2 + \lambda ||u_i||^2$$

Convex problem with closed-form

$$\hat{u}_i = (V_{\Omega^i}V_{\Omega^i} op + \lambda I)^{-1}V_{\Omega^i}^ op R_{\Omega^i}$$

Alternating least squares (ALS)

Randomly initialize U and V

- WHILE does not converge
 - $\begin{array}{l} \quad \forall i \in \mathcal{U}, \min_{u_i} || R_{\Omega^i} u_i^\top V_{\Omega^i} \top ||^2 + \lambda || u_i ||^2 \\ \quad \forall j \in \mathcal{I}, \min_{v_i} || R_{\Omega^j} v_j^\top U_{\Omega^j} \top ||^2 + \lambda || v_j ||^2 \end{array}$

MF for Implicit Feedback

- In real-world applications, we often observe more implicit feedback than explicit feedback.
- In fact, explicit feedback is sometimes considered implicit.
- Suppose user *i* watched 35% of movie *A* and 85% of movie *B*.

Does this mean that the user likes A more than B? If so, does it mean that the user likes A more than twice as much as B?

• The method we learned before is more appropriate for explicit feedback. Why?

Modelling Implicit Feedback

- Let's understand a more appropriate method
- Assume the binary interaction matrix *P*:

$$P = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}.$$

- That is, if user-*i* interact with item-*j*, than $P_{ij} = 1$, otherwise $P_{ij} = 0$.
- Now let C be a matrix of confidence regarding the interaction:

$$C = \begin{pmatrix} 0.85 & 0 & 0 & 0.34 & 0 & 0.98 \\ 0 & 0.37 & 0.10 & 0 & 0.63 & 0.01 \\ 0.45 & 0.42 & 0.43 & 0 & 0 & 0.23 \\ 0 & 0 & 0.26 & 0 & 0 & 0.88 \end{pmatrix}$$

Collaborative Filtering for Implicit Feedback

• Then we propose the following optimisation problem:

$$\mathsf{min}_{U,V}\sum_{i,j}C_{ij}(P_{ij}-u_i^\top v_j)^2 + \lambda ||u_i||^2 + \lambda ||v_j||^2$$

- Two main differences from previous MF method:
 - We need to account for the varying confidence levels
 - Optimization should account for all possible j, j pairs, rather than only those corresponding to observed data.
- We can use gradient descent to solve it.
- And ALS? By fixing V, can we find u_i ?
- 31 Recommender Systems EASSS 2023

Closed form

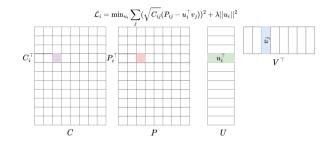
- Assume V being fix and let's find u_i .
- Then we need to minimize the following loss

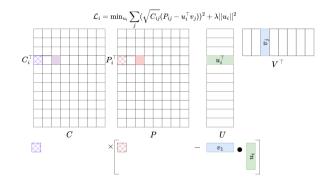
$$\mathcal{L}_{i} = \min_{u_{i}} \sum_{j} C_{ij} (P_{ij} - u_{i}^{ op} v_{j})^{2} + \lambda ||u_{i}||^{2}$$

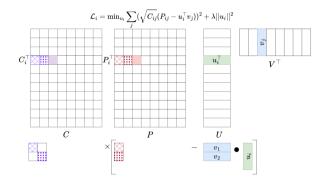
That is the same of:

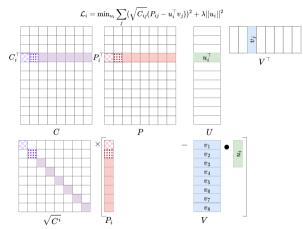
$$\mathcal{L}_i = \min_{u_i} \sum_j (\sqrt{C_{ij}} (P_{ij} - u_i^\top v_j))^2 + \lambda ||u_i||^2$$

Exercise: Find the closed form.









Closed form

• Therefore is the same of solving:

$$\mathcal{L}_i = ||\sqrt{C^i}P_i - \sqrt{C^i}Vu_i||^2 + \lambda + ||u_i||^2$$

• Taking the derivative

$$abla u_i = -2(\sqrt{C^i}V)^ op (\sqrt{C^i}P_i - \sqrt{C^i}Vu_i) + 2\lambda u_i$$

- Remind if D is diagonal $D = \sqrt{D} imes \sqrt{D}$ is trivial and $D = D^{ op}$
- Therefore, with just some algebraic derivations

$$u_i = (V^ op C^i V + \lambda I)^{-1} V^ op C^i P_i$$

RECAP: Autoencoder

- An autoencoder is a type of feed-forward neural network
- It is designed to reconstruct its input x_i ad output x_i
- To prevent trivial solutions, the network includes a bottleneck (or code) layer
 - Significantly smaller dimension than the input

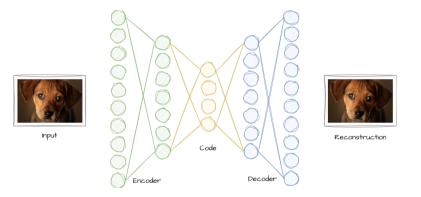
RECAP: Autoencoder

- An autoencoder is also composed by a encoder/decoder
- The encoder and the decoder have normally similar structure
- More formally: let *E*() be a encoder and *D*() be a decoder. Our optmization problem can be described as:

$$\min_{\mathcal{E}, \mathcal{D}} \sum_{i} ||x_i - \mathcal{D}(\mathcal{E}(x_i))||$$

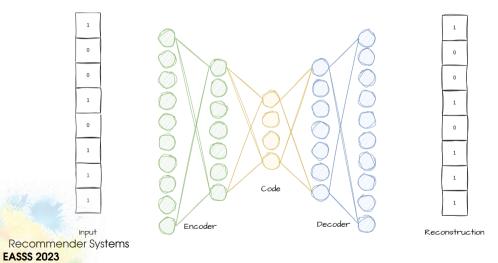
Autoencoder

37



Autoencoders for CF

38



Autoencoders for CF

- Autoencoders are frequently used for collaborative filtering.
- They are very accurate in predicting rankings.
- They can also be used to find clusters with the code.
- Empirical results show that the best architecture is often not very deep.
- What would be the shallowest autoencoder for Collaborative Filtering?

- EASE is the shallowest auto-encoder as possible
- It aims to solve the following problem

$$\min_B ||X - XB||^2 + \lambda ||B||^2$$
 s.t diag $(B) = 0$

- Why do we need the constraint diag(B) = 0?
- EASE has closed form solution! See here
- Is this a good method?
- 40 Recommender Systems EASSS 2023

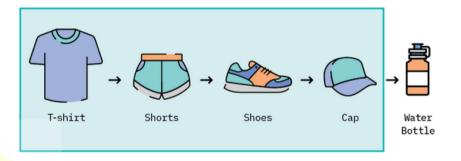
EASE Results

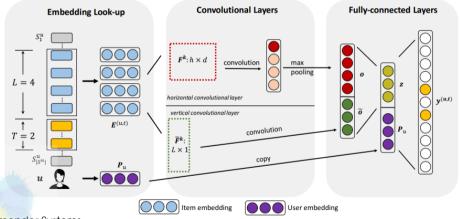
Table 1: Ranking accuracy (with standard errors of about 0.002, 0.001, and 0.001 on the *ML-20M*, *Netflix*, and *MSD* data, respectively), following the experimental set-up in [13].

(a) ML-20M	Recall@20	Recall@50	NDCG@100
popularity	0.162	0.235	0.191
EASER	0.391	0.521	0.420
$EASE^R \ge 0$	0.373	0.499	0.402
results reprod	uced from [13	3]:	
SLIM	0.370	0.495	0.401
WMF	0.360	0.498	0.386
CDAE	0.391	0.523	0.418
MULT-VAE PR	0.395	0.537	0.426
MULT-DAE	0.387	0.524	0.419
(b) Netflix			
popularity	0.116	0.175	0.159
EASER	0.362	0.445	0.393
$EASE^R \ge 0$	0.345	0.424	0.373
results reprod	uced from [13	3]:	
SLIM	0.347	0.428	0.379
WMF	0.316	0.404	0.351
CDAE	0.343	0.428	0.376
MULT-VAE PR	0.351	0.444	0.386
MULT-DAE	0.344	0.438	0.380
(c) MSD			
popularity	0.043	0.068	0.058
EASER	0.333	0.428	0.389
$EASE^R \ge 0$	0.324	0.418	0.379
results reprod	uced from [13	3]:	
SLIM	 did not finish in [13] — 		
WMF	0.211	0.312	0.257
CDAE	0.188	0.283	0.237
MULT-VAE PR	0.266	0.364	0.316
MULT-DAE	0.266	0.363	0.313

- Sequential recommendation is the task of predicting the next item that a user will interact
- There is extensive sequential recommendation algorithms
 - Markov chains
 - Recurrent neural networks (RNNs)
 - Long short-term memory (LSTM) networks
 - Embedding-base Neural Networks
- The models should learn patterns in a user's behavior over time







Triplets problem



Triplets problem

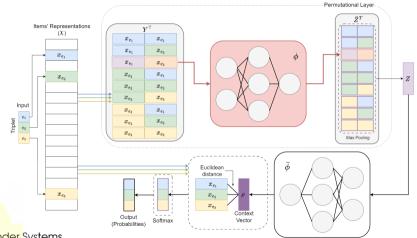


Triplets problem to Recommenders

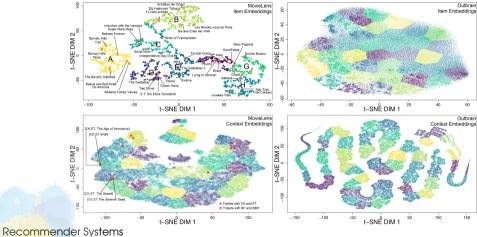


- The items we show to user can influence their decision
- Based on neuroscience
- Sometimes the position we show does not matter significantly
- Context embedding: summarizes the context of the recommendation
- Provide not just accurate recommendation but also interpretability

Care Model



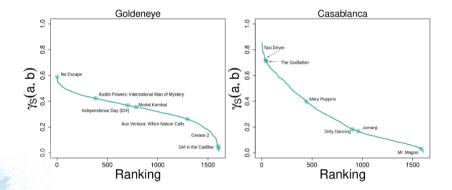
Care Model



EASSS 2023

48

Care Model



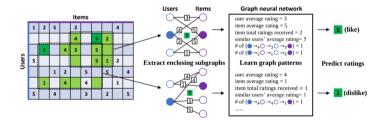
The Time Dimension in Recommendations

- Do you like the same things, morning and evenings?
- For example, the playlist recommendations on Spotify should change based on the time of day and day of the week.
 - Rarely do people have the same mood on Monday morning as they do on Friday evening.
- Taste and preferences change over time, so recommendations should adapt accordingly.
- The envoroment of RS is dynamic

Addressing the Cold-Start Problem

- Recommender systems typically require millions of interactions
- However, new systems often have limited interaction data available
- Attribute-based recommendations can provide valuable information
 - Normally less significant than interactions themselves

Transfer Learning in RS



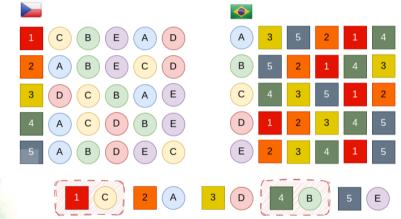
Tandem (Marriage) Problem



Greedy Czech Algorithm



Rogue Combination



Rogue Combination



Gale & Shapley Algorithm







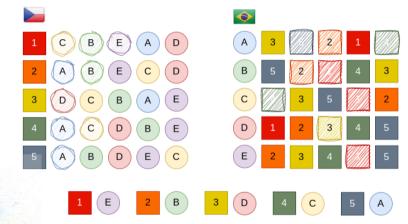








What about Recommender Systems?



What about Recommender Systems?



Tourists and Restaurants

- Restaurants has working times
- Tourists have visiting period
- Restaurants has limited number of tables
- Tourists has diet restrictions
- A new restaurant can be added to the item list
- A tourist might not have interactions for the visiting city

Recommendation are often a challenging task in a Tandem Problem scenario.

Multi-agents in RS

- Opportunity for research development
- More applied previous works with almost no theory
- Not restrict to the Tandem problem
- Not restrict to the user and items as agents

Modern matters in RS

- Fairness
 - Recommender systems have the potential to perpetuate or even amplify bias
 - Unequal treatment of different groups of users
- Filter Bubbles
 - Common problem on RSs that rely heavily on personalization
 - Recommendations that align with a user's pre-existing preferences
 - Negative consequences for both individual users and society

Modern matters in RS

- Challenging to evaluate
 - Lack of ground truth
 - Changes over the time
 - Diversity of user preferences
- Scalability
 - When terabytes of memory is not enough
 - Can result in increased computational costs and reduced performance

Modern matters in RS

- Privacy concerns
 - Recommender systems often rely on user data to provide accurate recommendations
 - Legislation (GDPR)
 - Lack of interpretability
- Dynamic preference
 - User preferences and item characteristics can be highly dynamic
 - Item availability
 - Difficult to provide accurate and up-to-date recommendations



Obrigado :) - Faculty of Information Technology